# Inconsistency of the Global Minkowski Space: A Geometric Critique

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Abstract

For over a century, global Minkowski spacetime ($\mathbb{M}^4$) has served as the foundational framework for four-dimensionalist ontologies and the ontological reification of the "block universe." This paper presents a strict geometric critique of this global structure by examining the non-commutative Lie algebra of the Poincaré group ISO(3, 1), specifically the non-vanishing commutator between Lorentz boosts and spatial translations ($[K_i, P_j] \neq 0$). By constructing a closed operational circuit of alternating boosts and displacements within a homogeneous coordinate matrix framework, we mathematically demonstrate the existence of a residual coordinate discrepancy—an effect defined here as *kinematic holonomy*. We show that this fundamental anholonomy forces a structural tearing of both spatial and temporal coordinates ($\Delta x \neq 0, \Delta t \neq 0$), rendering a globally seamless, rigid 4D coordinate grid an unambiguous algebraic impossibility. This structural breakdown is grounded in a scale-invariant "Andromeda paradox," demonstrating that global Minkowski space cannot consistently support localized, independent covariance without yielding ontological contradictions. By exposing global Minkowski space as an illicit coordinate artifact rather than a coherent physical backdrop, this work dismantles the geometric necessity of eternalism and restores the persistent 3D world as a mathematically rigorous model for physical reality.

## 1. Introduction

For over a century, the global Minkowski spacetime manifold ($\mathbb{M}^4$) has served as the foundational bedrock for four-dimensionalist ontologies, most notably the "block universe" model. Within these paradigms, past, present, and future are conceptualized as tenselessly co-existing within a rigid, statically extended geometric object. The primary justification for this radical metaphysical framework relies on an unjustified mathematical extrapolation: because local physical laws exhibit Lorentz covariance, it is assumed that the global physical arena supporting these laws must itself be a unified, globally Lorentz-covariant space. Consequently, traditional eternalism demands that for every event $P$ in this global manifold, there must exist a unique, unambiguous ontological state shared consistently by all observers (Appendix A).

In our previous work Yoon (2026), we challenged this global reification by examining the conceptual vulnerabilities of the Rietdijk-Putnam Andromeda paradox Rietdijk (1966); Putnam (1967); Penrose (1989). We argued that treating time as a globally extended dimension analogous to space introduces severe ontological anomalies, and we proposed a principal bundle framework as a more geometrically sound alternative. However, the standard four-dimensionalist defense frequently dismisses these paradoxes as harmless perspective effects—benign coordinate artifacts born from a passive shifting of reference frame hyperplanes, with no real structural consequence to the underlying block.

This paper presents a definitive geometric and algebraic refutation of the global Minkowski space by uncovering a fatal structural inconsistency inherent to its governing isometry group: the Poincaré group ISO(3, 1).[1] Mathematically, the Poincaré group is constructed as the semi-direct product of the Lorentz group and the four-dimensional translation group in space and time:

$$\mathrm{ISO}^{+}(3,1) = \mathbb{R}^{1,3} \rtimes \mathrm{SO}^{+}(3,1)\,.$$

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[1] We focus on the mathematical fallacies of the block universe only in the context of special relativity in this paper. On the one hand, we live in a world governed by matter and gravitation, and hence ultimately we will have to extend our arguments to general relativity, and even quantum mechanics. On the other hand, however, special relativity lies at the very local foundation of all modern physics Einstein (2026). Therefore, it is crucial to properly understand its ontological implications before we tackle broader and more general problems.

The semi-direct product ($\rtimes$) explicitly signifies that spatial displacements and pure Lorentz transformations do not cleanly decouple; the translation group is not a normal subgroup of the Poincaré group. Because the generators of Lorentz boosts ($K_i$) and spatial translations ($P_j$) do not commute ($[K_i, P_j] \neq 0$), a change of local reference frames applied at a single point rigidly and non-locally alters the coordinate layout of distant, spatially separated locations. This non-zero commutator mathematically guarantees that a pure kinematic boost cannot be purely localized or separated from its effect on the spatial grid.

By constructing a closed operational circuit of alternating boosts and displacements within a rigorous homogeneous coordinate matrix framework (Section 3), we demonstrate that this non-commutativity yields an inescapable coordinate tearing ($\Delta x \neq 0, \Delta t \neq 0$). This phenomenon, which we call *kinematic holonomy*, serves as the algebraic equivalent of parallel-transporting a vector around a closed curve on a curved manifold Misner, Thorne and Wheeler (1973); Wald (1984); tracking an observer through a closed circuit of kinematic actions fails to return the coordinate origin to its starting point.

We ground this structural breakdown by introducing a scale-invariant "Andromeda Street paradox" (Section 2), demonstrating that this geometric rupture occurs at any arbitrary spatial distance, down to the infinitesimal length scale. Ultimately, we analyze this contradiction through both passive and active operational views to show that the global Minkowski space cannot simultaneously support independent, localized physical covariance at every point on the manifold (Section 4). The global 4D block universe is revealed to be an untenable geometric hybrid—an illicit mathematical construction that attempts to stitch mutually incompatible, locally covariant tangent spaces into a single, rigid global backdrop. By exposing the global Minkowski space as a synthetic coordinate illusion rather than an objective physical stage, we dismantle the supposed geometric necessity of eternalism and restore the *persistent 3D world* as a mathematically rigorous option for physical reality (Section 5).

## 2. The Andromeda Street Paradox

The structural and ontological incoherence of a globally extended Minkowski space has long been hiding in plain sight, masked by the tacit assumption that the relativity of simultaneity is merely a passive "perspective trick." In a three-dimensional Euclidean space, a rotation changes an object's coordinate projection without altering its history or physical integrity; it is a passive reassignment of coordinates on a static, unified manifold. In contrast, treating the Lorentz-Einstein transformation as a mere perspective shift while simultaneously asserting that the four-dimensional Minkowski block ($\mathbb{M}^4$) is an objective physical reality forces an irreconcilable contradiction: it demands that a localized change of motion retroactively deletes or alters the temporal history of a distant observer.

To expose this flaw, we scale down the classic Rietdijk-Putnam Andromeda paradox Rietdijk (1966); Putnam (1967); Penrose (1989) into a localized, scale-invariant thought experiment: the *Andromeda Street paradox*. While eternalists frequently dismiss cosmic-scale hyperplane discrepancies as benign curiosities of hyper-planar math at a distance Petkov (2005); Rietdijk (1966), this miniaturized formulation demonstrates that the structural fracturing of a global 4D coordinate grid occurs at any arbitrary spatial scale. The resulting *micro-hyperbolic divergence* Yoon (2026) is not a harmless feature of eternalism that can be safely ignored at a short distance; it is a fatal self-contradiction. Several philosophers have noted the tension between relativistic geometry and our temporal experience, often debating whether these hyperplane shifts entail a frozen ontology Stein (1968, 1991); Savitt (2011); Price (1996, 2008).

Consider two observers, Alice ($A$) and Bob ($B$), initially at rest relative to one another on a persistent 3D spatial baseline—a local neighborhood road designated as "Andromeda Street." The street defines a mutual rest frame of reference. Alice and Bob are separated by a modest, finite spatial interval $\ell$ on the street (essentially, the $x$-axis). At a coordinate time $t < 0$, both observers are at rest and "share" an identical, globally flat plane of simultaneity. This is illustrated in Figure 1. The worldlines of Alice and Bob are represented by two vertical lines, $x = 0$ and $x = \ell$, respectively.[2] In this configuration, the local frame of Alice and that of Bob coincide with the global coordinate frame of Andromeda Street. In a typical eternalist view, they are one and the same thing. Or, alternatively, the "foliations" of Alice and Bob are considered to coincide with a foliation or "slice" (the street or the $x$ axis) of the global frame at time $t$ with $t < 0$.

[2]When a Minkowski diagram depicts a worldline extending continuously into the future, it smuggles in a deterministic, pre-existing structural commitment, making an assumption that the future is already known or ontologically existent. This is fundamentally absurd based on our everyday experience. We argued in Yoon (2026) that these four-dimensionalist arguments are fundamentally tautological. A Minkowski diagram should be considered valid only at a particular time and location ("here and now"), and everything else should be viewed as a potentially invalid extrapolation.

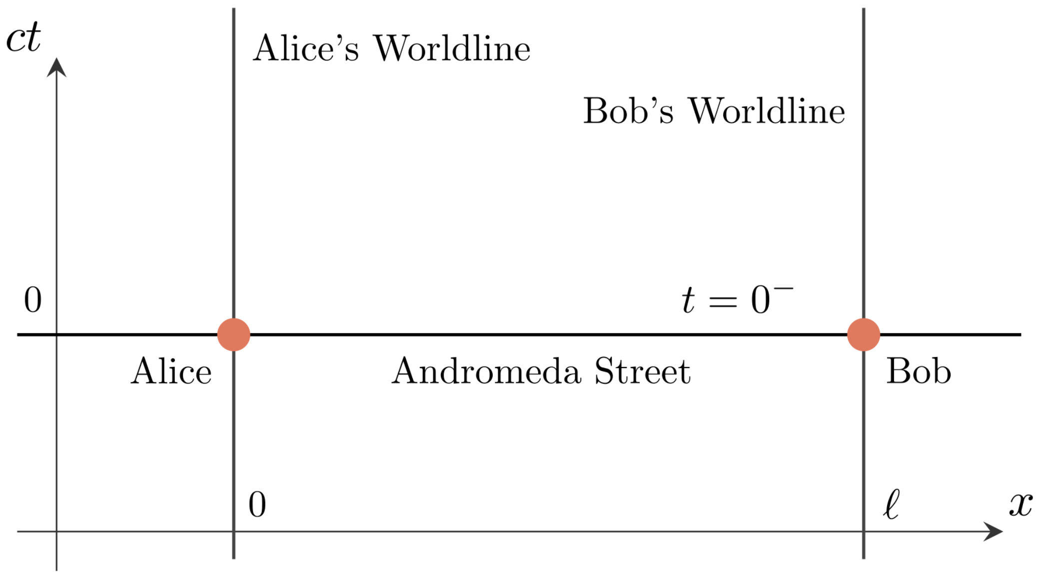


**Figure 1:** The Andromeda Street Paradox - The initial setup. Two "men on the street," Alice ($A$) and Bob ($B$), are at rest, at any time $t < 0$, relative to each other and with respect to a reference frame, "Andromeda Street." Alice and Bob are separated by a distance $\ell$ on this street along the $x$-axis, and they share an identical plane of simultaneity (the line labeled with $t = 0^{-}$). Note that we do not usually include light cones in the spacetime diagrams in this paper.

Now, let us introduce two localized, independent kinematic operations executed simultaneously with respect to the global frame of Andromeda Street at $t = 0$:

1. Alice undergoes an instantaneous, infinitesimal velocity boost $+v$ directed toward Bob along the $x$ axis.
2. Bob simultaneously undergoes an identical instantaneous, infinitesimal velocity boost $-v$ in the opposite direction toward Alice.

This is illustrated in Figure 2.[3]

## 2.1. Local Covariance vs. Global Consistency

Because the global Minkowski framework assumes that local Lorentz-Einstein transformations can be seamlessly extended into a rigid global backdrop (Appendix A), we must evaluate what happens at the boundaries of these localized boosts. This is illustrated in Figure 3. When Alice executes her boost $+v$, her local plane of simultaneity instantly tilts "to the left," as shown in the figure. To calculate the immediate coordinate status of Bob's location ($x = \ell$) within Alice's newly established local frame, we use the standard Lorentz-Einstein transformation equations:[4]

$$\Delta t_{B|A} = 0 - \gamma v \ell \approx -v\ell$$
$$\Delta x_{B|A} = \gamma \ell - 0 \approx \ell \,,$$

where $\gamma$ is the usual Lorentz factor, $1/\sqrt{1 - v^2}$. The notation "$B|A$" means that it is the property of $B$ (Bob) when viewed from the inertial reference frame $A$ (Alice).

[3] Although an idealized "instantaneous acceleration" is physically unrealizable, we model the operation by assuming it takes $\delta t$ seconds to reach steady motion, subsequently taking the mathematical limit $\lim_{\delta t \to 0}$. Even if the hyperplanes of simultaneity sweep continuously through time during a finite $\delta t$, the total accumulated temporal displacement across the spatial gap $\ell$ remains strictly identical once steady motion is achieved. Taking the sharp limit $\delta t \to 0$ isolates the structural discontinuity, proving it is baked directly into the kinematics of the Lorentz-Einstein transformation itself rather than the dynamics of acceleration.

[4] This refers to the more conventional term, the Lorentz-Einstein transformations, in this paper Einstein (2025). We use units where $c = 1$, hence the typical $1/c^2$ factor is absorbed into the expression.

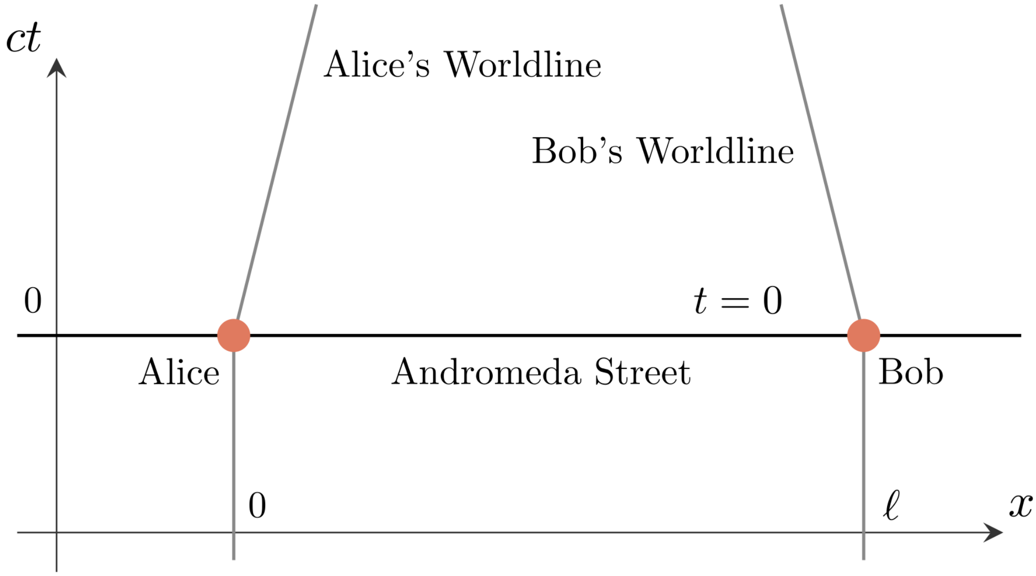


**Figure 2:** The worldlines of Alice and Bob. At $t = 0$, Alice and Bob start walking toward each other, with infinitesimal speeds $v$ and $-v$, respectively, with respect to the Andromeda Street reference frame. Their worldlines are tilted toward each other as well (looking into the future direction). Notice that these lines are "bent" at $t = 0$. While it is a common convention to designate the point where the coordinate axes meet as the origin $(0, 0)$, we do not follow such a convention.

Because the boost is directed toward Bob, Alice's local kinematic choice instantly shifts Bob's present into her coordinate past. According to the frame anchored at Alice, Bob's current slice of reality is already in her past. As far as Alice is concerned, the amount of coordinate time $\Delta t_{B|A}$ of Bob has been entirely "lost" (Figure 3).

At the same time as Alice's change of motion, Bob executes his independent boost $-v$ toward Alice. Evaluating Alice's location ($x = -\ell$) from within Bob's newly established local frame yields the reciprocal transformation:

$$\Delta t_{A|B} = 0 - \gamma(-v)(-\ell) \approx -v\ell$$
$$\Delta x_{A|B} = \gamma(-\ell) - 0 \approx -\ell \,.$$

Because Alice occupies the negative $x$-direction relative to Bob, his independent transformation instantly inclines his *now* "to the right," and Alice's local present jumps into his coordinate past. As far as Bob is concerned, Alice's time/duration $\Delta t_{A|B}$ in his frame has completely vanished from his local temporal continuity upon his boost relative to Andromeda Street, as indicated in the figure.

## 2.2. The Ontological Hyperbolic Rupture

If the global Minkowski space ($\mathbb{M}^4$) represents an objective, observer-independent physical block, these two localized transformations must be capable of being stitched into a single, cohesive global chart point-by-point. Instead, they expose an irreconcilable ontological contradiction. At the exact moment of their simultaneous boosts, Alice's frame instantly establishes that Bob's present resides in her coordinate past, while Bob's frame likewise establishes that Alice's present jumps into his coordinate past. Each observer encounters this instant and finite gap in their respective temporal continuity.

This ontological rupture is a direct mathematical consequence of the non-integrability of the localized "now" slices across a non-commuting isometry group. Although an instantaneous action is not practically possible in reality, this example shows that the observer's perspective on spatially separated events undergoes an instantaneous jump, or a discrete "fast forward," at the moment the observer changes their state of motion with respect to the global Minkowski space (Andromeda Street).

This ontological anomaly does not depend on astronomical distances; it happens even if Alice and Bob are separated by a meter, a millimeter, or an Angstrom. While the magnitude of the temporal drop $|\Delta t| = v\ell$ scales linearly with

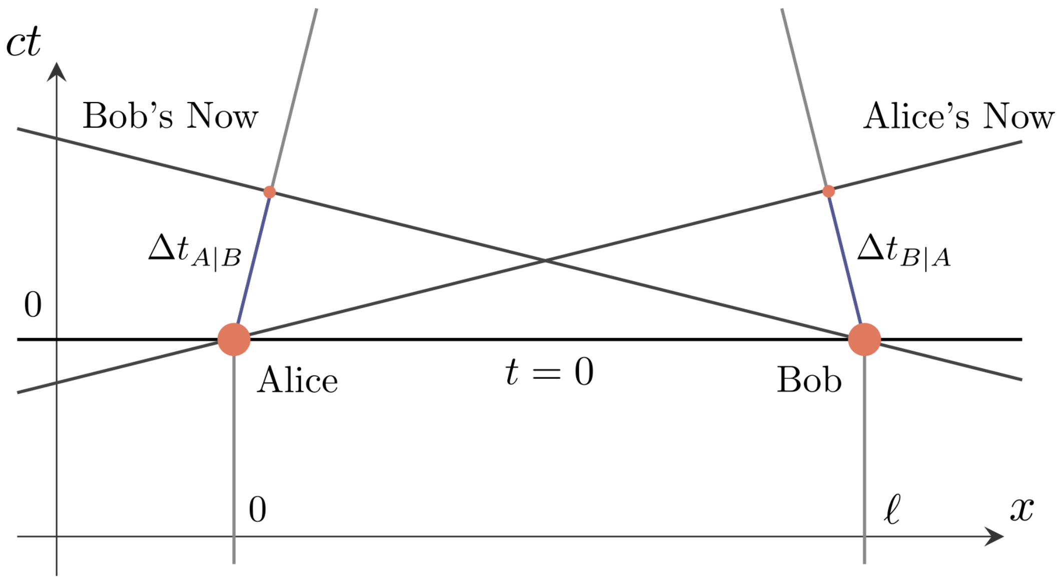


**Figure 3:** The hyperplanes of simultaneity ("now") for Alice and Bob. At $t = 0$, Alice and Bob start walking toward each other, with speeds $+v$ and $-v$, respectively. Their "nows" diverge at $t > 0$ in the Andromeda Street reference frame. Although its effect is rather small over a small distance, it nonetheless exposes the mathematical incongruity of the globally stitched coordinate frame.

distance, the structural rupture is purely geometrical and scale-invariant. The two local frames instantly fall into a flat contradiction with each other regarding the existential status of matter across the spatial gap $\ell \neq 0$ in the global hyperbolic manifold.[5]

This strongly supports our proposition that a globally-applied Lorentz-Einstein transformation cannot be interpreted as a smooth collection of localized physical facts. It is a rigid, holistic coordinate overlay that suffers from an immediate distributive failure. You cannot anchor global covariance at Alice without violating the physical integrity of Bob's local frame, nor can you anchor it at Bob without violating Alice's. There is no such thing as the "global Lorentz-Einstein transformation." The global Minkowski space structurally fractures the moment different physical points attempt to exercise local covariance independently.

To demonstrate that this neighborhood-scale rupture is not an isolated paradox, but the mathematical consequence of the underlying isometry group, we will formalize this operational circuit into a strict, closed loop on the Poincaré group Lie manifold in the following section.

### 2.3. Notes on Local Transformations

Before we continue, it is worth noting that this geometrical incoherence across the global manifold is not limited to the particular kinds of examples used in this paper. At the core of the issue is the fact that Lorentz-Einstein transformations at different locations cannot be simultaneously satisfied in the globally stitched Minkowski spacetime. For instance, Figure 4 shows an alternative kinematic configuration, in which Alice and Bob start walking in the same direction at $t = 0$ with the same velocity $v$. In this example, Alice's now and Bob's now are "parallel" to each other, yet they are still irreconcilable in the context of the global coordinate system.

Furthermore, although we focus on the two specific operations, Lorentz boost and translation, in this paper, the non-commutativity of Lorentz boosts and three-dimensional Euclidean rotations, for instance, also leads to ontological and geometric inconsistencies of the globalized Lorentzian manifold.[6]

[5] We stated, in our previous work Yoon (2026), that the Lorentz-Einstein transformations are only defined at the same point on a physical manifold and that they transform the vectors in the tangent space at the given point, and not the points/events on the manifold.

[6] The conclusions from this study will be communicated in an upcoming paper.

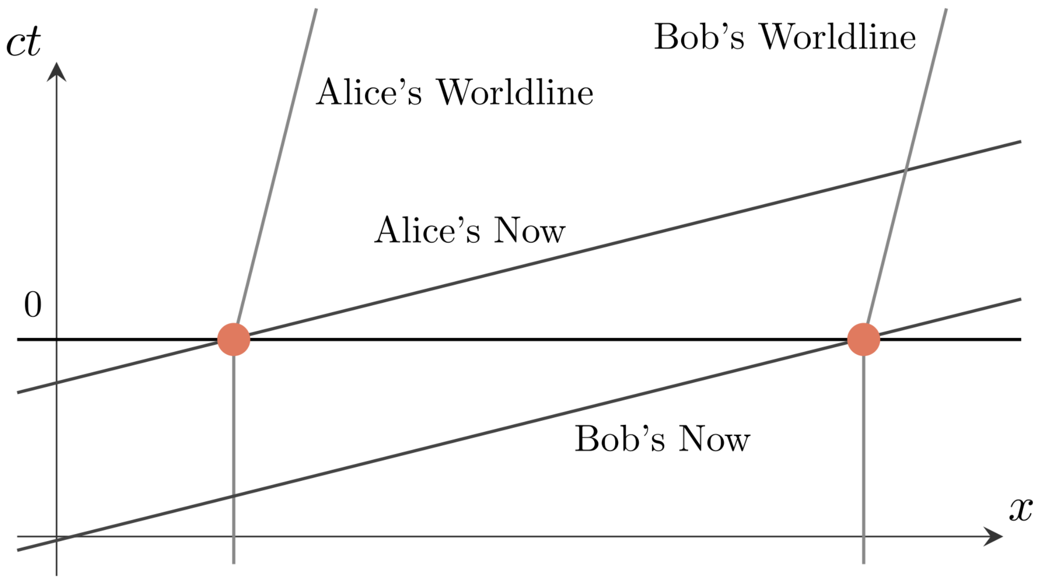


**Figure 4:** The hyperplanes of simultaneity for Alice and Bob in an alternative configuration, in which they start walking in the same direction at $t = 0$. Their "nows" do not align in this example either.

## 3. The Group-Theoretic Core: Kinematic Holonomy

To fully dismantle the century-long geometric bias that treats global Minkowski spacetime ($\mathbb{M}^4$) as a coherent, objective physical background, we must move beyond standard relational paradoxes and employ a strict operational test that is traditionally reserved as an analytical tool to study curved manifolds.

Critics frequently dismiss the relativity of simultaneity as a benign, passive mixing of space and time, analogous to how a 3D Euclidean rotation shuffles the $x$- and $y$-axes. However, by constructing a formal group-theoretic commutator loop on the Lie manifold of the Poincaré group ISO(3, 1) (the full symmetry group of special relativity), we demonstrate that this mixing induces a structural fracturing of the global coordinate system itself—a failure that has no parallel in flat Euclidean geometry.

The root of this inconsistency lies in the fundamental algebra of the Poincaré group generators Kibble (1961); Trautman (1970). While the generators of translations in space and time commute among themselves ($[P_a, P_b] = 0$, where $a, b = 0, 1, 2, 3$), the generators of Lorentz boosts ($K_i$, $i = 1, 2, 3$) and spatial translations ($P_j$, $j = 1, 2, 3$) satisfy the following non-vanishing commutation relation:

$$[K_i, P_j] = \eta_{ij} P_0 \,,$$

where $\eta_{ij}$ is the 4D Minkowski metric tensor and $P_0$ is the generator of temporal translations.[7] This non-zero commutator mathematically guarantees that a pure kinematic boost cannot be distributively separated from a spatial displacement. To evaluate the precise geometric significance of this non-commutativity in a global space, we model these transformations using an integrated matrix algebra.

### 3.1. Homogeneous Coordinate Matrix Framework

Because standard $2 \times 2$ (or $4 \times 4$) Lorentz matrices only map linear vector transformations, they fail to capture the affine nature of translations within a unified matrix multiplication layout. To resolve this, we map our $1 + 1$D spacetime transformations into a $3 \times 3$ *homogeneous coordinate representation*, appending a scale factor of 1 to our spacetime

[7] It is interesting to note that, within the mathematical framework of the Poincaré group, the temporal translation $P_0$ is not an independent generator. This reflects the fact that one cannot independently "move along the time axis."

column vectors. This framework allows us to embed both affine translations and linear boosts into a single, associative matrix algebra, providing a unified algebraic treatment of the Poincaré group's action.[8]

Let an arbitrary event $E$ be represented by the column vector:[9]

$$E = \begin{pmatrix} t \\ x \\ 1 \end{pmatrix}.$$

We define our two operational matrices for the passive perspective—where a translation shifts the coordinate grid by $a$ (effectively subtracting $a$ from the coordinates of the event) and a boost transitions to a frame moving at velocity $v$:

***The Passive Spatial Translation Matrix ($T_a$):***

$$T_a = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -a \\ 0 & 0 & 1 \end{pmatrix}.$$

***The Passive Lorentz Boost Matrix ($\Lambda_v$):***

$$\Lambda_v = \begin{pmatrix} \gamma & -\gamma v & 0 \\ -\gamma v & \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

## 3.2. The Operational Alternative Paths

We now construct two alternative operational sequences acting on the event $E$: Path 1 ($\Lambda_v T_a$) and Path 2 ($T_a \Lambda_v$).[10]

***Path 1: Spatial Translation followed by a Lorentz Boost ($\Lambda_v T_a$):***

We first shift the coordinate grid by $a$, and subsequently accelerate the grid to uniform velocity $v$, as illustrated in Figure 5 in the $T_a$-$\Lambda_v$ parameter space. Computing the combined transformation matrix $M_1$ yields:

$$M_1 = \Lambda_v T_a = \begin{pmatrix} \gamma & -\gamma v & 0 \\ -\gamma v & \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -a \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma v & \gamma v a \\ -\gamma v & \gamma & -\gamma a \\ 0 & 0 & 1 \end{pmatrix}. \tag{1}$$

Applying $M_1$ directly to our event vector $E$ reveals the transformed spatial and temporal coordinates for Path 1:

$$t_1 = \gamma(t - v(x - a)),$$
$$x_1 = \gamma((x - a) - vt).$$

***Path 2: Lorentz Boost followed by a Spatial Translation ($T_a \Lambda_v$):***

Conversely, we reverse the sequence of operations, accelerating the coordinate grid to velocity $v$ before executing the spatial displacement $a$. This is shown in Figure 6. Computing the combined transformation matrix $M_2$ yields:

$$M_2 = T_a \Lambda_v = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -a \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \gamma & -\gamma v & 0 \\ -\gamma v & \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma v & 0 \\ -\gamma v & \gamma & -a \\ 0 & 0 & 1 \end{pmatrix}. \tag{2}$$

Applying $M_2$ to our event vector $E$ reveals the transformed coordinates for Path 2:

$$t_2 = \gamma(t - vx),$$
$$x_2 = \gamma(x - vt) - a.$$

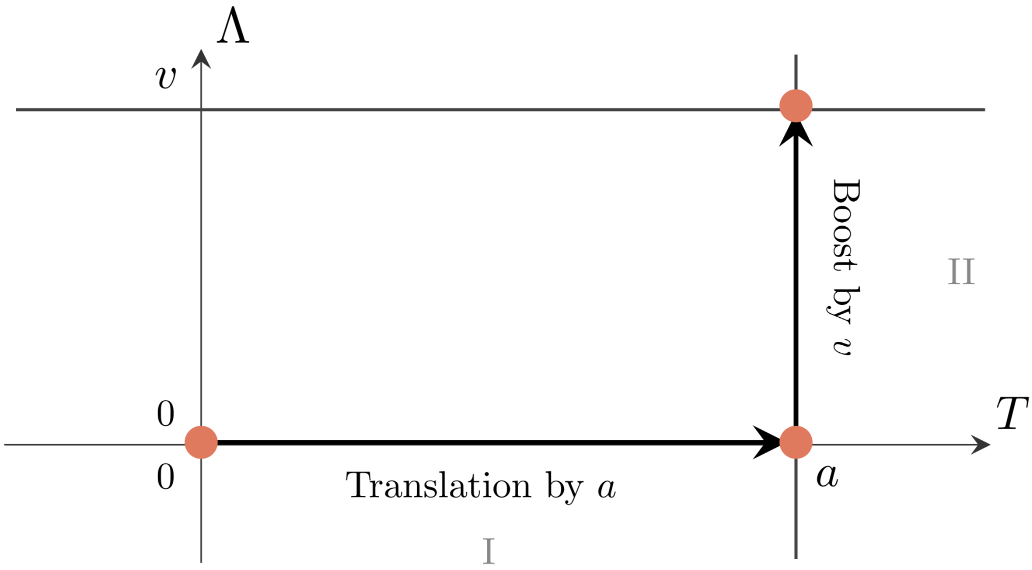


**Figure 5:** Path 1 on the Parameter Space: The axis $T$ (or $a$) represents a translation along the $x$-axis in the global Minkowski space physical manifold. The axis $\Lambda$ (or $v$) represents a Lorentz boost along the $x$-axis with velocity $v$. Path 1 consists of a translation by $a$ followed by a boost by $v$, terminating at the coordinate $(a, v)$ in the transformation parameter space.

### 3.3. The Commutator Tearing and Kinematic Holonomy

Figure 7 shows both of these paths in a single parameter space diagram. Path 1 and Path 2 terminate at the same final coordinates in the $v$-$a$ parameter space ($X$) essentially by definition. Their geometric effects on the physical manifold, however, are vastly different due to the non-commutativity of spatial translations and Lorentz boosts.[11] This precise distinction has been the primary source of misunderstanding surrounding global Minkowski structures for over a century.

Because a global Minkowski manifold superficially mirrors the properties of its local tangent spaces, it has been tacitly assumed that the linear properties of the tangent space apply seamlessly to the global flat manifold without rigorous validation (Appendix C). We expose this visual and conceptual fallacy by strictly applying the non-commutative rules of the underlying isometry group.

Figure 8 depicts the resulting coordinate frame after the sequential transformations of Path 1. The original Cartesian coordinate grid with origin $O$ has been shifted and tilted into the new frame $(t_1, x_1)$. Its coordinate origin $O_1$ corresponds to the event $t_1 = 0, x_1 = 0$. Conversely, the origin of the original grid, $O$, is represented in the new frame by $(t_1, x_1) = (\gamma v a, -\gamma a)$, which can be verified by applying $M_1$ (Eq. 1) to the original origin vector $(0, 0, 1)^T$.[12] Crucially, the combined sequence of a translation and a boost creates an irreversible temporal shift.

Likewise, Figure 9 shows the coordinate frame after the consecutive passive transformations of Path 2. While this new coordinate frame appears qualitatively similar to that of Path 1, their respective origins map to entirely different coordinates. The original origin $O$ is represented in this frame by $(t_2, x_2) = (0, -a)$, verified by applying $M_2$ (Eq. 2) to $(0, 0, 1)^T$. Following Path 2, the original observer views the origin as spatially displaced but completely devoid of any temporal shift.

If global Minkowski space were an objectively seamless, rigid 4D manifold, the sequence in which an observer mathematically re-labels their coordinate axes through passive transformations would be entirely trivial. The grid

[8] Corresponding to $5 \times 5$ matrices in a full 4D spacetime. Homogeneous coordinates are widely used, *inter alia,* in projective geometry and computer animation.

[9] As we emphasized in Yoon (2026), an event or point on a manifold is not a vector. However, the global Minkowski space formulation relies on a *trivial isomorphism* that treats world points as vectors.

[10] Due to the right-to-left nature of matrix multiplication, the sequence of operations is written in the reverse order of application.

[11] Clearly, this discrepancy holds true for any non-trivial configuration where both $v \neq 0$ and $a \neq 0$.

[12] Without loss of generality, we assume $a > 0$ and $v > 0$. All illustrations and associated comments rely on this convention.

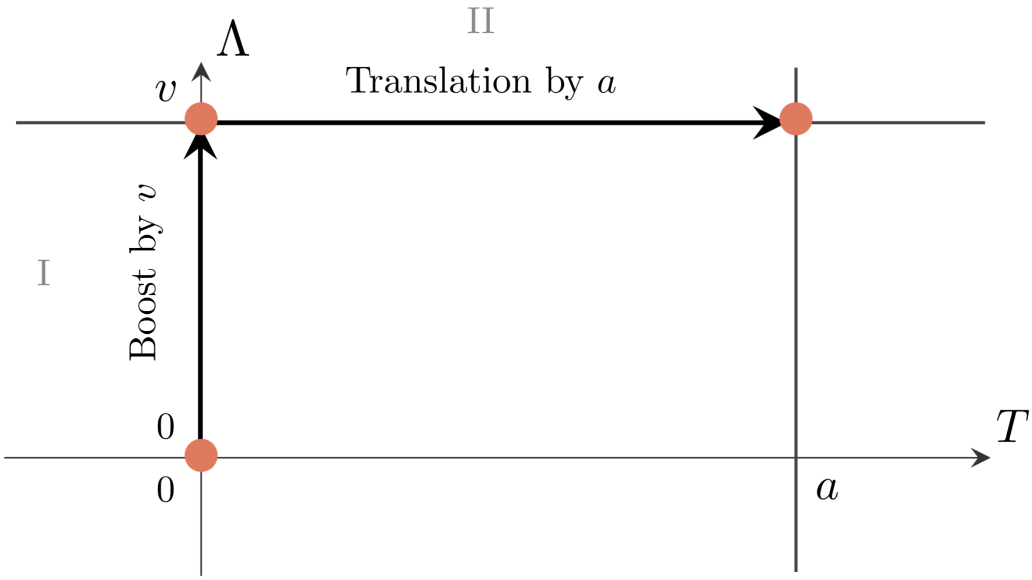


**Figure 6:** Path 2 on the Parameter Space: First a boost by $v$, followed by a translation by $a$. This path ends up at the same point as Path 1, sharing the identical final parameter values $(a, v)$.

transformations would commute perfectly, yielding an identical final frame regardless of the operational order. To measure the exact structural discrepancy generated by the non-commutative algebra of the Poincaré group, we compute the direct difference matrix ($\Delta M = M_1 - M_2$):

$$\Delta M = \begin{pmatrix} \gamma & -\gamma v & \gamma v a \\ -\gamma v & \gamma & -\gamma a \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} \gamma & -\gamma v & 0 \\ -\gamma v & \gamma & -a \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & \gamma v a \\ 0 & 0 & (1-\gamma)a \\ 0 & 0 & 0 \end{pmatrix}.$$

Evaluating this difference matrix against the coordinate origin vector $(0, 0, 1)^T$ yields the explicit, irreversible coordinate tearing equations forced out by the Poincaré algebra:

$$\Delta t = \gamma v a\,, \tag{3}$$
$$\Delta x = (1-\gamma)a\,, \tag{4}$$

where neither value vanishes insofar as $a \neq 0$ and $v \neq 0$.

The resulting coordinate divergence is illustrated in Figure 10. Neither the time nor space axes of the $O_1$ and $O_2$ systems coincide with one another. In fact, these discrepancies grow monotonically as a function of spatial displacement $a$ and boost velocity $v$.

In standard differential geometry, curvature or torsion is detected when a vector is parallel-transported around a closed loop on a manifold and fails to return to its original orientation—a phenomenon known as holonomy. Equations (3) and (4) demonstrate the precise operational equivalent on the Poincaré Lie manifold: *kinematic holonomy*.

Just as a vector tracked around a curved sphere accumulates a geometric phase shift due to surface curvature, a coordinate origin tracked through an alternating circuit of displacements and boosts returns fundamentally fractured and disoriented. The parameter space of special relativity is inherently *anholonomic*. Because changing the order of basic kinematic operations yields a persistent spatial gap $\Delta x = (1-\gamma)a$ and an absolute temporal drop $\Delta t = \gamma v a$, the global Minkowski coordinate grid cannot close. The assumption that $\mathbb{M}^4$ represents a static, globally coherent 4D "solid block" is mathematically and structurally untenable under the strict algebraic rules of its own isometry group.[13]

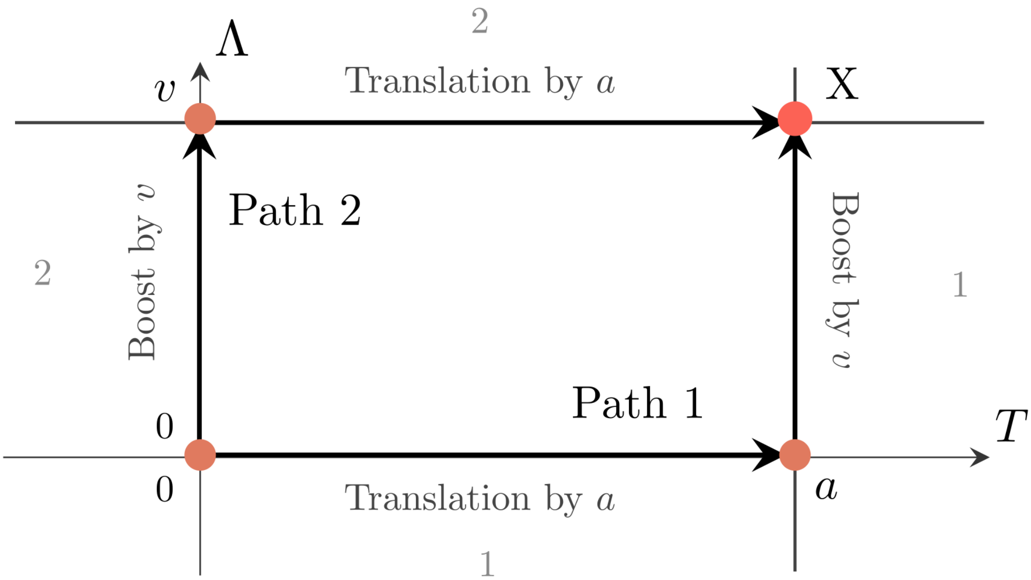


**Figure 7:** Both Paths on the Parameter Space: Path 1 consists of (Translation by $a$) → (Boost by $v$). Path 2 consists of (Boost by $v$) → (Translation by $a$). Although the two paths appear identical in parameter space, their actual effects on the physical manifold differ fundamentally because translations and Lorentz boosts do not commute.

### 3.4. Discussion

Although the mathematical framework necessary to uncover the structural failures of global Minkowski space has been available for decades, this geometric inconsistency has slipped through an academic crack for over a century. To encapsulate our main argument, let us trace a single closed loop of consecutive coordinate transformations rather than comparing distinct alternative paths.

The circuit in Figure 11 consists of four sequential coordinate transformations: $\Lambda_{-v}T_{-a}\Lambda_v T_a$. As previously emphasized, closing this loop in the parameter space is a geometric illusion. After executing these four passive transformations, the coordinate frame fails to return to its initial untransformed identity state ($a = 0, v = 0$). This serves as explicit proof that one cannot maintain a holonomic coordinate system across global Minkowski space.

Using the identical matrix formalism, the original frame returns from the four-transformation circuit as the following system:

$$M_{\text{loop}} = \Lambda_{-v}T_{-a}\Lambda_v T_a = \begin{pmatrix} 1 & 0 & \gamma v a \\ 0 & 1 & (1-\gamma)a \\ 0 & 0 & 1 \end{pmatrix}.$$

This results in a clean relation to our previous path divergence matrix $\Delta M$:

$$M_{\text{loop}} = I + \Delta M\,,$$

where $I$ is the identity matrix. Because the combined operations do not collapse back to the identity matrix, the anholonomic character of a globally patched Minkowski space is explicitly exposed. Strictly speaking, one cannot construct a consistent, unbroken coordinate chart over any finite region in a global Lorentzian manifold (Appendix C). The singular exception occurs when either $a = 0$ or $v = 0$.[14]

[13] Interestingly, we can view the well-studied Twin Paradox, at least in part, as a direct physical manifestation of the anholonomic nature of global Minkowski spacetime. We briefly discuss this geometric view in Appendix B.

[14] This implies that one can only preserve a rigid global Minkowski space, like a static block, if all physical motion ceases entirely—amounting to a universe completely frozen in time as well as space.

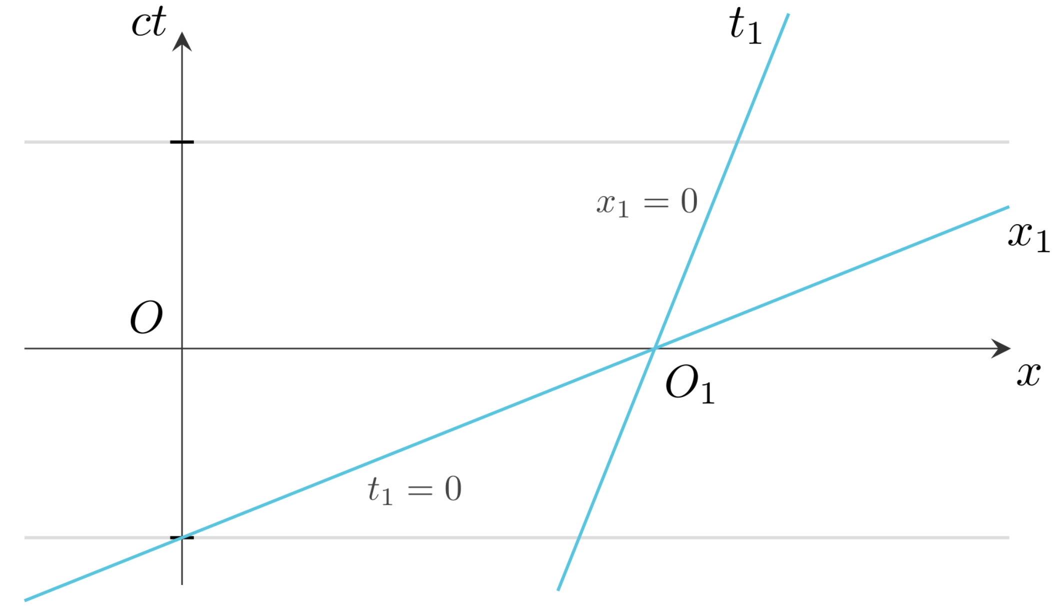


**Figure 8:** The resulting coordinate grid upon transformations of Path 1. $O$ represents the origin of the original coordinate system $(t, x)$, and $O_1$ that of the final system $(t_1, x_1)$. Light propagates along the diagonal directions at every point.

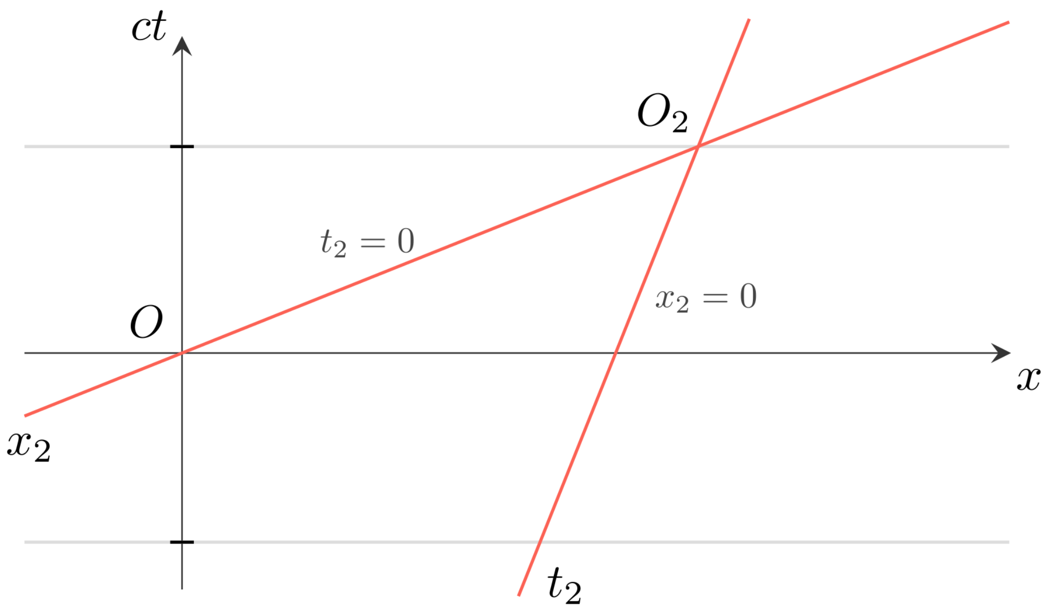


**Figure 9:** The coordinate grid upon transformations following Path 2. $O$ represents the origin of the untransformed coordinate system $(t, x)$, whereas $O_2$ is the origin of the twice-transformed coordinate system $(t_2, x_2)$.

The resulting discrepancies in time and space coordinates mirror our path deviations:

$$\Delta t_{\text{loop}} = \gamma v a \,,$$
$$\Delta x_{\text{loop}} = (1 - \gamma) a \,,$$

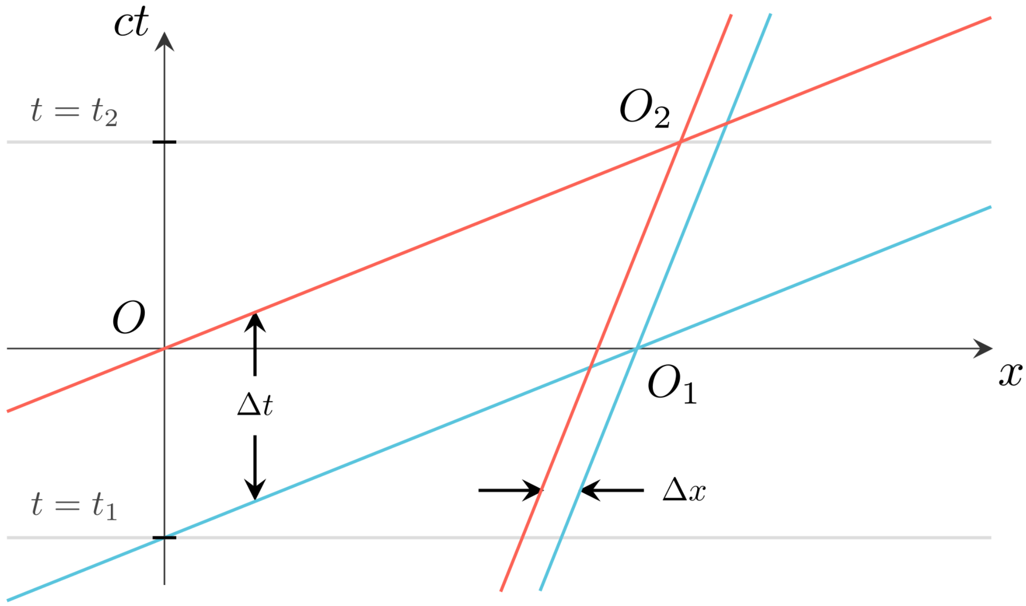


**Figure 10:** The structural mismatch between the transformed coordinate frames of Path 1 and Path 2. The coordinate discrepancies are designated by the spatial and temporal displacements $\Delta t$ and $\Delta x$ along the axes of the original grid. Neither value is constant; both depend strictly on $a$ and $v$.

which are non-zero for all $v \neq 0$ and $a \neq 0$. This cumulative discrepancy is not a geometrically invariant physical quantity; its spacetime interval,

$$\Delta s^2 = \Delta t^2 - \Delta x^2 = 2a^2(\gamma - 1)\,,$$

is neither zero nor Lorentz invariant.[15] It is purely an artifact of an ill-defined, globalized coordinate overlay.

Ultimately, $t$ and $x$ are not holonomic coordinates in a global Minkowski space, despite their superficial resemblance to the purely spatial coordinates of flat Euclidean geometry. Furthermore, the space is not flat in an absolute sense due to the inherent curvature present within the Lie manifold of its own isometry group (Appendix C).

Strictly speaking, no two disjoint points on a Lorentzian manifold—no matter how small their metric separation—can be seamlessly integrated under the global rules of the Poincaré group. In practice, however, this structural fracture scales down to negligible values when $v \ll c$ and $a \ll c\delta t$, where $\delta t$ represents the threshold of local temporal tolerance or experimental uncertainty in a given local environment. We demonstrated in Yoon (2026) that ontological coherence can be approximately preserved as long as the spatial domain remains well within the local *ontological radius*, given by $c^2\delta t/v$.

This insight rescues the practical utility of local Lorentzian coordinate charts. By treating the flat Minkowski metric as a strictly localized approximation valid only within restricted spatiotemporal neighborhoods—fully utilizing the exponential map as is standard practice for curved manifolds—we resolve the paradox of why our everyday experience appears uniformly flat. This explains why the global Minkowski illusion has persisted for so long: it matches our local intuition. The fallacy is fundamentally identical to that of our ancestors who believed the Earth was flat for millennia, misled by the undeniable fact that the planetary surface is, indeed, *approximately flat* on a localized human scale.[16]

[15] We exclude the trivial origin of parameter space ($a = 0, v = 0$) from representing a valid physical reality. A universe stripped of the capacity for relative displacement or velocity is a dead world where relativity ceases to have any functional meaning.

[16] We previously modeled this using a terrestrial velocity baseline of $10^3$ m/s Yoon (2026). High-velocity orbital systems, such as GPS satellites, operate outside this immediate local ontological zone. Consequently, their onboard clocks cannot maintain structural synchronicity with the surface without continuous, explicit relativistic corrections.

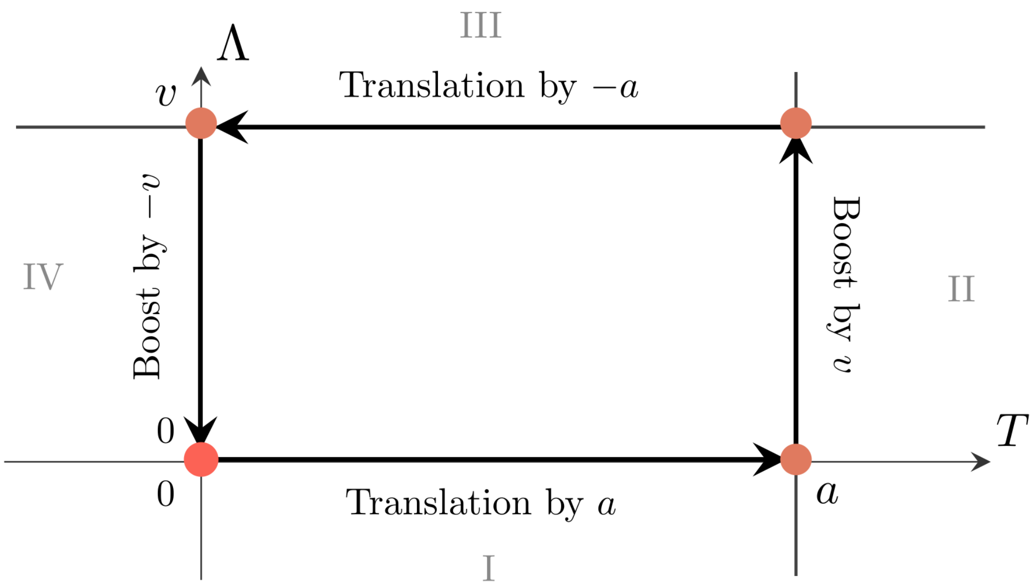


**Figure 11:** A closed transformation loop on the parameter space consisting of the sequence: (Translation by $a$) → (Boost by $v$) → (Translation by $-a$) → (Boost by $-v$).

## 4. Untenability of the Block Universe

A common counterargument from traditional four-dimensionalists is that the coordinate tearing calculated in equations (3) and (4) is merely a benign artifact of coordinate definitions. The conventional textbook defense argues that the spatial translation parameter $a$, when executed within a moving reference frame, naturally undergoes length contraction relative to the baseline rest frame. Therefore, the traditionalist claims, if one manually updates the translation transformation parameter from $a$ to $\gamma a$ to account for this physical contraction, the geometric loop will close seamlessly on paper, leaving no traceable physical discrepancy.

However, subjecting this defense to rigorous group-theoretic scrutiny reveals that it commits a fundamental category error, forcing the eternalist into a deeper geometric dilemma: the *rigid coordinate fallacy*.

### 4.1. The Rigid Coordinate Fallacy

If coordinate transformations in special relativity are purely passive—representing nothing more than a pencil-and-paper re-labeling of a pre-existing, static four-dimensional block—then the parameters of the transformation group must remain rigid, global constants. They cannot dynamically alter their values based on the historical sequence of operations or depending on where exactly the transformations are initiated. The coordinate values must remain globally covariant under the fundamental symmetry group of special relativity, ISO(3, 1) Brown (2005).

If we maintain strict mathematical rigidity and demand that the spatial displacement parameter $a$ remains identical across both alternative paths, we are trapped by the spatial mismatch:

$$\Delta x = (1 - \gamma)a \neq 0 .$$

The global 4D grid flatly fails to preserve physical lengths across alternative passive sequences, as demonstrated in Section 3.

On the other hand, if the eternalist attempts to rescue the spatial grid by manually rewriting Path 2—insisting that the moving observer must translate by a dynamically adjusted, length-contracted coordinate step $\gamma a$—the spatial axes can be forced to artificially align based on the prior boost transformation, but the temporal coordinate now bears the

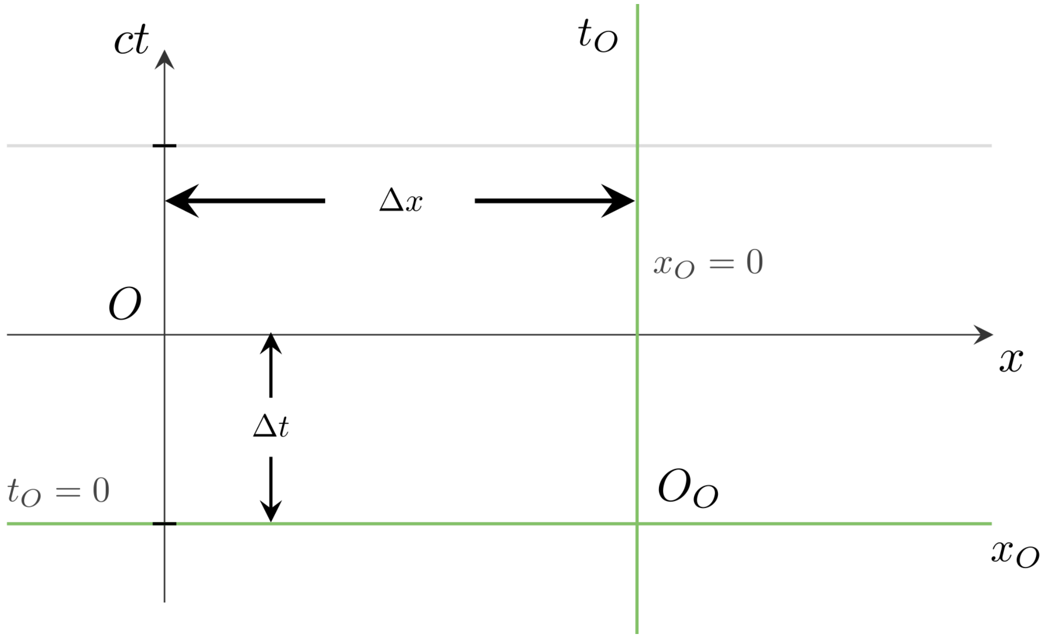


**Figure 12:** The net geometric shift of a coordinate frame traversing a closed loop relative to the original untransformed coordinate system. The resulting axes remain parallel to the original frame but suffer fixed displacements $\Delta t$ and $\Delta x$ that remain non-vanishing for all non-trivial operations.

full weight of the algebraic failure. Swapping $a$ for $\gamma a$ in Path 2 transforms the difference matrix into:

$$\Delta M_{\text{adjusted}} = \begin{pmatrix} \gamma & -\gamma v & \gamma v a \\ -\gamma v & \gamma & -\gamma a \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} \gamma & -\gamma v & 0 \\ -\gamma v & \gamma & -\gamma a \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & \gamma v a \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Evaluating this adjusted difference matrix against the origin vector $(0, 0, 1)^T$ yields a pure, unmitigated temporal rupture:

$$\Delta t = \gamma v a\,.$$

Even after artificially patching the spatial gap at a point, the temporal drop stubbornly persists. The eternalist has not resolved the contradiction; they have merely shifted a spatial mismatch into an absolute temporal drop. The coordinate grid still refuses to close. To eliminate this residual temporal gap on a rigid grid, an observer would be forced to manually adjust their clocks based on their historical velocity path for every single point across the global manifold.

This result presents an insurmountable challenge to the global block universe model. It forces the admission that coordinates on a global 4D manifold are not objective constants of a static arena, but are history-dependent, path-variable parameters. The passive global grid cannot be consistently closed without violating its own structural definition. Ultimately, it is mathematically impossible to define a holonomic coordinate system on the global $t$-$x$ Minkowski space that remains consistent with the Poincaré group.

## 4.2. The Active Operational View

The failure of the passive coordinate grid forces us to shift our analysis to the *active operational perspective*. Here, the Lie group parameters $(a, v)$ are no longer treated as passive re-labelings of a global background; instead, they are isolated as frame-independent, active physical instructions executed on localized matter. The parameter $a$ represents a real physical displacement (e.g., firing an engine to move a localized apparatus), and the velocity $v$ represents a real, physical change of motion via acceleration.

If the global Minkowski block exists as an objective, observer-independent physical container, it must be capable of consistently hosting a closed circuit of these active physical operations. Yet, the non-commutative structure of the

Poincaré algebra dictates that an active circuit forces an irreversible physical divergence. If a localized physical system executes Path 1 (displacing by $a$, then boosting to $v$) versus Path 2 (boosting to $v$, then displacing by $a$), the system accumulates a genuine spatial drift and an absolute temporal desynchronization in the physical world.

This exposure completely invalidates the eternalist defense. What the passive view attempts to sanitize as a mere "relativity of perspective," the active operational view exposes as a structural fracturing of physical reality. A global, rigid coordinate grid cannot geometrically survive the active, distributed execution of its own symmetry group. The structural tearing is real, and it proves that the unified 4D block is a mathematical fiction.

### 4.3. Geometric Reality vs. Dynamic Reality

The geometric crisis exposed by *kinematic holonomy* is not merely that alternative operational paths yield differing numerical parameters on a grid. Rather, the profound ontological failure lies in the fact that independent historical paths strip observers of a shared, coherent sense of perspectival location and orientation, even when occupying the exact same geometric event.

This path-dependent perspective triggers an immediate ontological collapse for the traditional block universe. The eternalist framework dictates that every event in the 4D block must possess a unique, determinate ontological state. Yet, because observers reaching an identical event may bring entirely different path histories, their respective alignments in both space and time diverge. Consequently, the block universe forces a physical absurdity: two different observers, while standing at the exact same microscopic spot, possessing the exact same instantaneous velocity, and looking through the exact same instruments, are structurally bound to entirely different configurations of past and future.

If we accept the block universe ontology, what is already an unalterable historical fact for one observer remains an unformed future event for another. If reality is defined by a global 4D manifold constructed from meshes of mutually incompatible local hyperplanes, then a physical object's historical path leaves a permanent, irreconcilable fracture in the objective fabric of reality at the very point it stands. As we demonstrate throughout this paper, however, such a globalized reality does not exist. The very concept of a rigid spacetime that globally obeys the local symmetries of special relativity is a mathematical hallucination.

The structural mismatch of the local presence in the global block universe also highlights a profound tension between fundamental kinematics and the psychology of temporal experience. As Huw Price has compellingly argued from an "Archimedean" standpoint, common-sense intuitions regarding an objective present, a directional flow, and a fundamental temporal asymmetry are not intrinsic features of physical geometry, but are rather anthropocentric projections born from our localized cognitive and thermodynamic constraints Price (1996, 2012); Rovelli (2017).

However, the geometric reality of kinematic holonomy introduces an even deeper crisis for the eternalist who seeks to use this psychological illusion to defend a rigid global backdrop. If the experiential "now" is merely a subjective projection mapped onto a pre-existing 4D stage, that projection must at least be structurally coherent for converging observers. Instead, the algebraic reality of the Poincaré group dictates that the very mechanism of physical motion systematically twists and tears these localized frameworks when reconciling the perspectives of different observers. Observers cannot even agree on a shared subjective coordinate grid. This demonstrates that the traditional block universe fails from both directions: it lacks the structural flatness required to support an objective global present, and it lacks the geometric consistency required to smoothly synchronize independent paths of experiential time (Appendix B).

## 5. Conclusion: An Alternative 3D Perspective

Einstein's special relativity is ultimately about local Lorentz symmetry. On a global scale, the flat world of special relativity is formally governed by the Poincaré group, which includes the extended invariance under spatial translations. Minkowski's geometric formulation of special relativity in 4D hyperbolic space was originally created as a mathematical tool for enforcing the Lorentz covariance on dynamics at a point on the physical manifold Minkowski (1952).[17] Over time, however, Minkowski's 4D geometry has taken over the entire manifold, ultimately giving birth to the block universe ontology. This paper has shown that this monolithic "block" is fundamentally incompatible with the Poincaré group, the isometry group of special relativity.

The ultimate inconsistency in the reified block universe model—one that has been systematically overlooked in the philosophy of time for over a century—is the profound failure of global structural coherence. The block model asserts the objective existence of a singular, rigid, four-dimensional "stage" shared simultaneously by all physical observers.

[17]Minkowski space is a linear space. We claimed, in reference Yoon (2026), that global Minkowski space was a mathematical extrapolation through the trivial isomorphism, which had no justification in physics.

However, as this paper has demonstrated, the strict Lie group algebra of special relativity proves that this stage is fundamentally unglobalizable.

The global Minkowski space ($\mathbb{M}^4$) is an illicit mathematical construction born from mistaking a passive coordinate convenience for an active physical container. Because the generators of spatial translations and Lorentz boosts do not commute ($[K_i, P_j] \neq 0$), attempting to stitch independent, locally covariant reference frames into a globally rigid grid triggers an inescapable coordinate tearing ($\Delta x \neq 0, \Delta t \neq 0$). As demonstrated by the scale-invariant Andromeda Street paradox, this *kinematic holonomy* fractures the coordinate layout at any arbitrary distance, down to the microscopic neighborhood scale. The moment independent physical observers attempt to exercise local covariance simultaneously, the global 4D grid rips apart under its own algebraic constraints.

To be abundantly clear, the geometric critique presented in this work does not constitute a dogmatic, blanket refutation of all four-dimensional physical structures. It remains entirely possible that the universe possesses a fundamentally four-dimensional physical character. Rather, this refutation is directed strictly and uncompromisingly at the reification of an un-gauged, globally extended Minkowski space as a seamless backdrop. If four-dimensionalism is to survive the mathematical reality of kinematic holonomy, it must adapt. It can no longer rely on a static, rigid block universe; it must instead transition to a highly localized, dynamic gauge framework where a spacetime connection field ($A_\mu$) is introduced to actively absorb and track these path-dependent coordinate discrepancies.

From the perspective of Occam's razor, however, this structural breakdown strongly points toward a far more elegant and parsimonious alternative. Why spend immense mathematical and metaphysical effort reifying an unobservable, globally extended 4D block—and then constructing complex gauge connections simply to patch its global tears—when the exact same empirical facts are perfectly preserved by a localized fiber bundle geometry? Yoon (2026)

True physical Lorentz covariance is not a global property of an empty container; it is a strictly local kinematic property of physical matter in motion. By moving the 4D Minkowski structure out of the global background and interpreting it strictly as a local *tangent space* attached to a persistent, dynamic three-dimensional base manifold ($M^3$), all global geometric ruptures instantly vanish. This tangent bundle framework preserves all the predictive power of special relativity without the structural inconsistencies forced by global reification. The coordinate tears both in space and time no longer signify a ripping of the fabric of existence; they simply describe how the internal clocks and reference frames of localized 3D physical objects dynamically behave as they interact.

Once we strip away the synthetic coordinate illusion of a global 4D backdrop, the dogmatic necessity of eternalism completely dissolves. By demonstrating the mathematical impossibility of the global Minkowski space, we close the escape hatches of the block universe and firmly restore the persistent, changing "3D world" as a mathematically rigorous and fully viable option for physical reality Balaguer (2021); Wüthrich (2010); Ellis and Drossel (2020).

## A. Ontological and Mathematical Definitions of the "Block Universe"

The *block universe*—the eternalist ontology wherein past, present, and future are deemed "equally real"—is frequently invoked in literature as a self-evident metaphysical consequence of special relativity Rietdijk (1966); Putnam (1967); Petkov (2005). Yet, it is rarely afforded a precise, mathematically rigorous definition Savitt (2011). It operates largely as a conceptual "Bigfoot"—omnipresent in philosophical discourse but structurally elusive when subjected to formal geometric scrutiny. To execute a valid refutation, we must establish a concrete mathematical definition of the target ontology based on its pervasive implicit descriptions.

We define the traditional, un-gauged block universe as a physical model satisfying two strict mathematical axioms:

### Axiom 1: The Principle of Global Existential Completeness

Physical reality is completely and exhaustively modeled by a single, maximal, inextendible four-dimensional Lorentzian manifold ($\mathcal{M}, \eta_{\mu\nu}$). For any two events $P, Q \in \mathcal{M}$, the ontological truth value of their physical existence is identical and invariant:

$$\text{Exist}(P) \equiv \text{Exist}(Q) \equiv \text{True}\,.$$

The manifold $\mathcal{M}$ is a set-theoretic whole given statically "all at once." Temporal progression is relegated to a passive, internal coordinate parameter rather than an active, ontologically generative process.

### Axiom 2: The Rigidity of the Global Slicing Map

The global manifold $\mathcal{M}$ is assumed to be globally hyperbolic, admitting a trivial topological decomposition $\mathcal{M} \cong \mathbb{R} \times \Sigma$, where $\Sigma$ is a smooth three-dimensional spatial Cauchy surface representing a global "Now." For any local coordinate chart associated with an inertial observer, the transition functions mapping the local plane of simultaneity to distant regions must be globally integrable (holonomic). This integrability must preserve the unique ontological identity of matter at every point $P \in \mathcal{M}$, completely free of path-dependency.

By establishing this rigid, un-gauged global grid as the mathematical core of eternalism, our critique in Section 3 demonstrates that Axiom 2 is algebraically incompatible with the fundamental symmetry group of special relativity, ISO(3, 1). Because the non-commutative structure of the Poincaré group forbids the global integration of these local hyperplanes across alternative path histories, the entire conceptual block collapses.

## B. The Twin Paradox as a Manifestation of Kinematic Holonomy

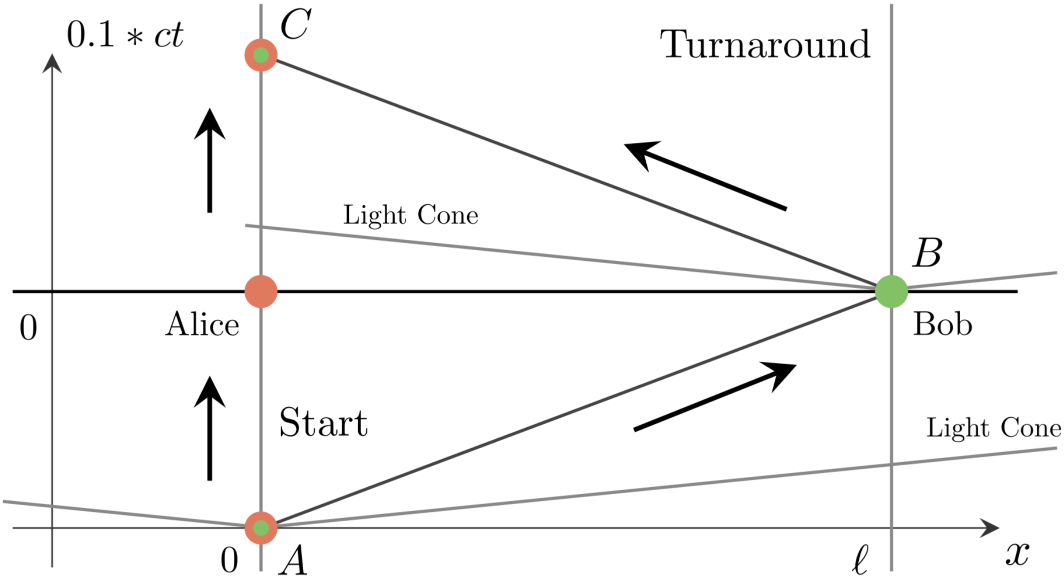


**Figure 13:** The Twin Paradox as Kinematic Holonomy: Alice remains at rest while Bob executes a closed loop in space. The accumulated temporal desynchronization between their local clocks is the physical manifestation of the non-commutativity of the Poincaré generators. Note that, in the particular scaling used here, all light cones have a slope of $0.1$ or $-0.1$.

The Twin Paradox is conventionally framed as a biological or physical anomaly arising purely from localized acceleration, or from the geometric structural differences between straight and curved worldlines within a pre-existing, static four-dimensional block. However, its true underlying engine is entirely group-theoretic. It serves as a direct, empirical manifestation of the non-commutativity of translations and Lorentz boosts ($[K_i, P_j] \neq 0$) within the Poincaré group ISO(3, 1).

Consider the standard formulation: the traveling twin (Bob) departs a persistent spatial origin, accelerates to a relativistic velocity, turns around, and returns to the stationary twin (Alice). Conventionally, eternalists point to the mismatch in their respective proper times ($\Delta\tau$) as absolute proof that Bob simply carved a shorter path through a static, pre-existing 4D spacetime container.

When evaluated through the lens of kinematic holonomy, however, the paradox exposes a far deeper crisis: the total failure of a global, passive coordinate system. Traditionalists routinely conflate the path-dependent nature of proper time $\tau$ with the assumed rigidity of a background global coordinate time $t$. But because Bob's journey constitutes an active, completely closed physical loop in space ($\oint dx = 0$), a holonomic global coordinate system demands that the net coordinate time displacement around the circuit must likewise close ($\oint dt = 0$).

Yet, because the active physical operations of spatial translation and Lorentz boosting do not commute, executing this closed operational circuit forces a real, irreversible coordinate divergence at the point of reunion:

$$\Delta t = \oint dt = \gamma v a \neq 0 \,.$$

The temporal desynchronization accumulated between the twins is not a passive consequence of traversing an elongated track in a pre-existing 4D block. It is the exact physical accumulation of a *kinematic phase shift* along a localized time axis Penrose (2004). Bob returns younger because his historical sequence of non-commuting active kinematic actions generated a real, path-dependent temporal drop relative to Alice. The Twin Paradox does not validate global eternalism; it physically unmasks the fact that coordinate time is anholonomic and path-dependent, proving that a seamless, global "now" grid cannot be consistently stitched across moving bodies.

To see this operationally, let Alice and Bob be at rest at event $A$ (at coordinate time $t = t_A^-$ and location $x = x_A$) with respect to the baseline laboratory frame.[18] At time $t = t_A$, Bob initiates active acceleration to enter his outward journey in the $+x$ direction. Alice remains at the same spatial location throughout Bob's trip, meaning her frame of reference continuously coincides with the baseline background grid. As Bob undergoes his outward journey, his local hyperplane of simultaneity undergoes a hyperbolic rotation relative to Alice.

Upon his turnaround and reversal at event $B$, the non-commutativity of the active boost and the spatial displacement prevents his local reference frame from ever re-synchronizing with Alice's without an artificial, manual intervention. This physical desynchronization is the exact operational realization of the kinematic holonomy loop derived in Section 3.

From the kinematic holonomy perspective maintained in this paper, a static and eternal 4D block is mathematically untenable. The twins age differently upon their ultimate reunion at event $C$ because the specific sequence of physical motions they executed generated a real, path-dependent temporal phase shift that cannot be sanitized or erased within any globally covariant framework. The Twin Paradox, far from proving the block universe, serves as its ultimate empirical refutation, demonstrating that time is a localized, non-integrable evolutionary parameter rather than a global coordinate dimension.

## C. The Non-Flatness of the Poincaré Parameter Space

The foundational justification for the physical reification of the global Minkowski spacetime ($\mathbb{M}^4$) rests on its purported mathematical flatness. Because the Riemann curvature tensor calculated from the Minkowski metric $\eta_{\mu\nu}$ vanishes identically everywhere ($R^{\mu}{}_{\nu\rho\sigma} = 0$), physicists and philosophers have historically concluded that the global manifold is a trivial, passive, and structurally seamless geometric backdrop Misner et al. (1973); Wald (1984).

This conclusion completely overlooks a profound mathematical distinction: while the base manifold $\mathbb{M}^4$ is metrically flat under passive coordinate descriptions, the parameter space of its governing isometry group—the Poincaré group ISO(3, 1)—is fundamentally non-flat, non-Abelian, and possesses its own intrinsic group-theoretic curvature. The metric flatness of $\eta_{\mu\nu}$ merely guarantees that parallel-transporting a standard geometric vector along a closed path within a *single, undisturbed reference frame* yields no orientation shift Wald (1984). However, when an observer actively executes a closed circuit of kinematic transformations involving alternating spatial displacements and velocity boosts, they are tracking a loop through the parameter space of the Lie group itself.

Because $[K_i, P_j] = \eta_{ij} P_0$, the structure constants of the Poincaré algebra act as an effective kinematic curvature. The non-zero temporal drop ($\Delta t = \gamma v a$) and the spatial tearing ($\Delta x = (1-\gamma)a$) derived herein act as the exact algebraic equivalent of a geometric holonomy deficit. Minkowski space is locally flat but kinematically curved.

The traditional 4D block universe is an illusion born from a deep category error: mistaking the vanishing Riemann curvature of a passive coordinate grid for the total structural flatness of physical kinematics. Once the curved, anholonomic reality of the Poincaré group space is recognized, the reification of a rigid global Minkowski space becomes mathematically untenable.

## Acknowledgments

I am deeply grateful to Professor Huw Price (University of Cambridge) for graciously endorsing my preprint on arXiv, despite our fundamentally different views on the ontology of time and my status as an independent researcher.

[18]In Appendix A of Yoon (2026), we pointed out the subtle visual bias that the 2D Minkowski diagram creates. For three-dimensionalists, this figure should be viewed as a collection of horizontal snapshots at distinct times, flowing from bottom to top in the direction of temporal evolution.

## CRediT authorship contribution statement

**Harry Yoon:** The author confirms being the sole contributor of this work and has approved it for publication. During the preparation of this manuscript, the author utilized the Gemini large language model for conceptual clarification, background research, and drafting assistance. Following this process, the author personally reviewed, critically revised, and edited the content to ensure technical accuracy, mathematical rigor, and scientific integrity. The author takes full responsibility for the final version of the manuscript and its conclusions..

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